Way to Discriminate between Mesons and Glueballs for $I = 0, J^{PC} = even^{++}$ Unflavored Hadrons

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Abstract

Based on the general analysis of branching ratio of two pseudoscalar meson channels, discriminants between mesons and glueballs for $I = 0, J^{PC} = even^{++}$ unflavored hadrons with mass between 1.2 GeV and 2.9 GeV are suggested. Known $I = 0, J^{PC} = 2^{++}, f_2(1525)$ particle is discriminated as a typical meson. The way to discriminate new $I = 0, J^{PC} = even^{++}$ unflavored hadrons is discussed.

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Any $I = 0, J^{PC} = even^{++}$ unflavored hadron X can be a meson composed of light quark and anti-quark, a hybrid composed of light quark and anti-quark and gluon, a two gluon glueball or a meson composed of heavy quark and anti-quark. If the mass of X is lighter than 2.9 GeV, the possibility of being a heavy quarkonium should be ruled out. One may discriminate these possibilities with each other by the decay behavior of this hadron. In general X can decay into two pseudoscalar mesons and the possible decay modes are: $\pi^+\pi^-, \pi^0\pi^-, K^+K^-, K_SK_S, K_LK_L, \eta\eta, \eta\eta', \eta'\eta'$. The effective Hamiltonian of such kind of decay modes can be described generally as [1]

$$H_{eff} = \frac{f}{m_X^{J-!}} X_{\mu\nu\ldots\lambda\sigma}(\partial_1^\mu - \partial_2^\mu)(\partial_1^\nu - \partial_2^\nu)\ldots(\partial_1^\lambda - \partial_2^\lambda)(\partial_1^\sigma - \partial_2^\sigma)P_1P_2, \tag{1}$$

where P_1 and P_2 describe peudoscalar mesons in final state and $X_{\mu\nu\ldots\lambda\sigma}$ with J subscripts describes the particle X of spin J and satisfies constraints: $X_{\mu\nu\ldots\lambda\sigma}$ is fully symmetric in $\nu\mu\ldots\lambda\sigma$; $g^{\mu\nu}X_{\mu\nu\ldots\lambda\sigma} = 0$ and $\partial^\mu X_{\mu\nu\ldots\lambda\sigma} = 0$.

The partial width $\Gamma(P_1P_2)$ of decay mode $X \to P_1 + P_2$ can be expressed as

$$\Gamma(P_1P_2) = \eta\alpha\frac{k^{2J+1}}{m_X^{2J}}, \tag{2}$$

where k is the decay momentum in the center of mass system, m_X is the mass of X particle, α is the dimensionless effective coupling constant

$$\alpha = \frac{f^2}{4\pi} \tag{3}$$

η is a numerical coefficient which takes the value of

$$\eta = \frac{8^J (J!)^2}{2(2J+1)!}, \tag{4}$$

If X is a meson composed of light quark and anti-quark, the flavor structure of effective interaction can be described generally as

$$\begin{aligned} H_{eff} = \; & \tfrac{1}{m_X^{J-1}}[f_1 Tr(XPP) + f_2 Tr(X) Tr(PP) \\ & + f_3 Tr(XP) Tr(P) + f_4 Tr(X) Tr(P) Tr(P)], \end{aligned} \tag{5}$$

where X and P are 3×3 flavor matrices which have the form

$$X = X_J \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & A \end{pmatrix}. \tag{6}$$

where $A = 1$ or $A = -2$ means X is a singlet or an octet of flavor $SU(3)$ group respectively, A takes other values means X belongs to a mixing state $\mathbf{1}+\mathbf{8}$ of flavor $SU(3)$ group.

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \alpha\eta + \beta\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \alpha\eta + \beta\eta' & K^0 \\ K^- & \overline{K}^0 & -\sqrt{2}\beta\eta + \sqrt{2}\alpha\eta' \end{pmatrix}. \tag{7}$$

where

$$\alpha = \frac{\cos\theta - \sqrt{2}\sin\theta}{\sqrt{6}}, \beta = \frac{\sin\theta + \sqrt{2}\cos\theta}{\sqrt{6}}. \tag{8}$$

θ is the mixing angle of $\mathbf{1}+\mathbf{8}$ for pseudoscalar mesons.

The values of coupling constants f_1, f_2, f_3 and f_4 depend on the dynamical mechanism of specific decay modes, and are independent to each other. The first term of the effective interaction involving f_1 describes the Zweig allowed decay while the other terms involving f_2, f_3 and f_4 describe the Zweig suppressed terms. Thus f_2, f_3 and f_4 are expected to be much smaller than f_1 and their contributions to the decay width should be negligible in comparison with f_1. The flavor structure of the effective interaction of X meson decaying into two pseudoscalar mesons can be described simply as

$$H_{eff} = \frac{f_1}{m_X^{J-1}} Tr(XPP), \tag{9}$$

The dimensionless effective coupling constant α_{eff} can be expressed as

$$\alpha_{eff} = G(P_1P_2)\alpha_1, \tag{10}$$

where the coefficient $G(P_1P_2)$ can be calculated from above effective interaction as in Table 1.

Table 1 Coefficient $G(P_1P_2)$ of decay mode $X \to P_1 + P_2$ for meson X.

Modes	$G(P_1P_2)$	Modes	$G(P_1P_2)$
$\pi^0\pi^0$	2	K^+K^-	$(1+A)^2$
$\pi^+\pi^-$	4	$\eta\eta$	$8(\alpha^2+A\beta^2)^2$
K_SK_S	$\frac{(1+A)^2}{2}$	$\eta\eta'$	$16\alpha^2\beta^2(1-A)^2$
K_LK_L	$\frac{(1+A)^2}{2}$	$\eta'\eta'$	$8(\beta^2+A\alpha^2)^2$

If one introduces $R(\frac{P_1P_2}{P_3P_4})$

$$R(\frac{P_1P_2}{P_3P_4}) = \frac{\alpha_{eff}(P_1P_2)}{\alpha_{eff}(P_3P_4)}, \tag{11}$$

two discriminants of X being a pure meson composed by light quark and anti-quark can be obtained as

$$R(\frac{\eta\eta}{K^+K^-}) = \frac{2(\alpha^2-\beta^2)^2}{R(\frac{K^+K^-}{\pi^+\pi^-})} + \frac{8(\alpha^2-\beta^2)\beta^2}{\sqrt{R(\frac{K^+K^-}{\pi^+\pi^-})}} + 8\beta^4, \tag{12}$$

and

$$R(\frac{\eta\eta}{K^+K^-}) + R(\frac{\eta\eta'}{K^+K^-}) + R(\frac{\eta'\eta'}{K^+K^-}) = 2 + \frac{1}{R(\frac{K^+K^-}{\pi^+\pi^-})} - \frac{2}{\sqrt{R(\frac{K^+K^-}{\pi^+\pi^-})}}. \tag{13}$$

These two discriminants are independent of whether X ia a pure singlet, belongs to a octet or is a mixing state of $\mathbf{1}+\mathbf{8}$ representation of flavor $SU(3)$ group.

¿From these relations one may find that if $R(\frac{K^+K^-}{\pi^+\pi^-}) \geq 0.25$, then $R(\frac{\eta\eta}{K^+K^-}) + R(\frac{\eta\eta'}{K^+K^-}) + R(\frac{\eta'\eta'}{K^+K^-}) \leq 2.0$. So, if experimental data shows that $R(\frac{\eta\eta}{K^+K^-}) + R(\frac{\eta\eta'}{K^+K^-}) + R(\frac{\eta'\eta'}{K^+K^-}) >> 2.0$, the possibility of X being a pure meson state should be ruled out.

If X is a hybrid composed of light quark, anti-quark and a gluon, the flavor structure of effective interaction can be described in the same way as that of the meson case, so it is difficult to distinguish hybrid from meson by its behavior of these decay modes.

If X is a glueball composed of two gluons, the flavor structure of effective interaction can be described generally as

$$H_{eff} = \frac{1}{m_X^{J-1}}[f_2X_JTr(PP) + f_4X_JTr(P)Tr(P)], \tag{14}$$

The values of coupling constants f_2 and f_4 depend on the dynamical mechanism of specific decay modes, and are independent to each other. Both terms of the effective interaction will give contribution to specific decay modes.

If the term involving f_2 dominates the decay, the coefficient $G(P_1P_2)$ can be calculated from above effective interaction and the results are listed in Table 2.

Table 2 Coefficient $G(P_1P_2)$ of decay mode $X \to P_1 + P_2$ for glueball X with f_2 term dominates.

Modes	$G(P_1P_2)$	Modes	$G(P_1P_2)$
$\pi^0\pi^0$	2	K^+K^-	4
$\pi^+\pi^-$	4	$\eta\eta$	2
K_SK_S	2	$\eta\eta'$	0
K_LK_L	2	$\eta'\eta'$	2

Four discriminants of X being a glueball with f_2 term dominate can be obtained as

$$R(\frac{K^+K^-}{\pi^+\pi^-}) = 1, \tag{15}$$

$$R(\frac{\eta\eta}{K^+K^-}) = \frac{1}{2}, \tag{16}$$

$$R(\frac{\eta\eta'}{K^+K^-}) = 0, \tag{17}$$

and

$$R(\frac{\eta'\eta'}{K^+K^-}) = \frac{1}{2}. \tag{18}$$

If the term involving f_4 dominates the decay, the coefficient $G(P_1P_2)$ can be calculated from the above effective interaction as shown in Table 3.

Table 3 Coefficient $G(P_1P_2)$ of decay mode $X \to P_1 + P_2$ for glueball X with f_4 term dominates.

Modes	$G(P_1P_2)$	Modes	$G(P_1P_2)$
$\pi^0\pi^0$	0	K^+K^-	0
$\pi^+\pi^-$	0	$\eta\eta$	$18\sin^4\theta$
K_SK_S	0	$\eta\eta'$	$36\sin^2\theta\cos^2\theta$
K_LK_L	0	$\eta'\eta'$	$18\cos^4\theta$

This term leads to only three decay modes: $\eta\eta$, $\eta\eta'$ and $\eta'\eta'$. In general, both terms of interaction will give contributions to the decay, the effective coupling constant should be the sum of these two terms. Then, in general, three discriminants of glueball should be changed as:

$$R(\frac{K^+K^-}{\pi^+\pi^-}) = 1, \tag{19}$$

$$R(\frac{\eta\eta}{K^+K^-}) \geq \frac{1}{2}, \tag{20}$$

and

$$R(\frac{\eta\eta}{K^+K^-}) + R(\frac{\eta\eta'}{K^+K^-}) + R(\frac{\eta'\eta'}{K^+K^-}) \geq 1. \tag{21}$$

$R(\frac{P_1P_2}{P_3P_4})$ canbe obtained from partial widths of these two decay modes by

$$R(\frac{P_1P_2}{P_3P_4}) = \frac{\Gamma(P_1P_2)k_{P_3P_4}^{2J+1}}{\Gamma(P_3P_4)k_{P_1P_2}^{2J+1}}, \tag{22}$$

and α and β are known functions of the mixing angle θ of pseudoscalar mesons, thus all quantities in (12), (13), (19), (20) and (21) can be obtained directly from experiments. These five relations can be treated as a set of discriminants for whether a $I = 0, J^{PC} = even^{++}$ unflavored hadrons with mass between 1.2 GeV and 2.9 GeV is a meson, a hybrid or a two gluon glueball.

It is important to note that for the case of $R(\frac{K^+K^-}{\pi^+\pi^-}) \neq 1.00$, the possibility of X being a pure glueball should be ruled out. But if X is a pure light quarkonium or a hybrid with light quark and antioquark, the values of $R(\frac{\eta\eta}{K^+K^-})$ and $R(\frac{\eta\eta}{K^+K^-}) + R(\frac{\eta\eta'}{K^+K^-}) + R(\frac{\eta'\eta'}{K^+K^-})$ should satisfy equations (12) and (13) respectively. If the experimental values of $R(\frac{\eta\eta}{K^+K^-})$ and $R(\frac{\eta\eta}{K^+K^-}) + R(\frac{\eta\eta'}{K^+K^-}) + R(\frac{\eta'\eta'}{K^+K^-})$ are larger than the corresponding values expected from (12) and (13) for the pure meson or hybrid case, X should be a mixing state of glueball and light quarkonium or hybrid. Thus (12), (13), (19), (20) and (21) can be adopted to judge whether a particle is a pure glueball, a pure meson or hybrid, or a mixing state of glueball and meson or hybrid unambiguously.

Above discriminants can be described in Fig 1 and Fig 2. If the values of $R(\frac{\eta\eta}{K^+K^-})$ and $R(\frac{\eta\eta}{K^+K^-}) + R(\frac{\eta\eta'}{K^+K^-}) + R(\frac{\eta'\eta'}{K^+K^-})$ are laid on the solid lines in Fig 1 and Fig 2, X should be a pure light quarkonium or a haybrid. If the values of $R(\frac{\eta\eta}{K^+K^-})$ and $R(\frac{\eta\eta}{K^+K^-}) + R(\frac{\eta\eta'}{K^+K^-}) + R(\frac{\eta'\eta'}{K^+K^-})$ are laid on the dashed lines in Fig 1 and Fig 2, X should be a pure glueball. If the values of $R(\frac{\eta\eta}{K^+K^-})$ and $R(\frac{\eta\eta}{K^+K^-}) + R(\frac{\eta\eta'}{K^+K^-}) + R(\frac{\eta'\eta'}{K^+K^-})$ are laid above the solid lines and not on the dashed lines in Fig 1 and Fig 2, X should be a mixing state of light quarkonium or haybrid and glueball.

One well-known particle $f_2(1525)$ is an $I = 0, J^{PC} = 2^{++}$ unflavored hadron. Its branching ratios of two pseudoscalar decay modes are $Br(K\overline{K}) = (88.8 \pm 3.1)\%$, $Br(\eta\eta) = (10.3 \pm 3.1)\%$ and $(\pi\pi) = (0.82 \pm 0.15)\%$, respectively [2]. Thus one gets $R(\frac{K^+K^-}{\pi_+\pi_-}) = (290 \pm 38)$. According to the above discriminants, the possibility of $f_2(1525)$ being a pure glueball should be ruled out. If $R(\frac{\eta\eta}{K^+K^-}) = 0.224$, $f_2(1525)$ should be a pure meson or a hybrid; if $R(\frac{\eta\eta}{K^+K^-}) \gg 0.224$, $f_2(1525)$ should be a mixing state. From above data one gets $R(\frac{\eta\eta}{K^+K^-}) = (0.36 \pm 0.11)$, so it leads to the conclusion that $f_2(1525)$ is a pure meson or hybrid.

One new unflavored particle $f_J(2220)$ was observed in radiative decay of J/ψ meson [3]. The spin of $f_J(2220)$ is $J = 2$ or 4. The combined branching ratios of

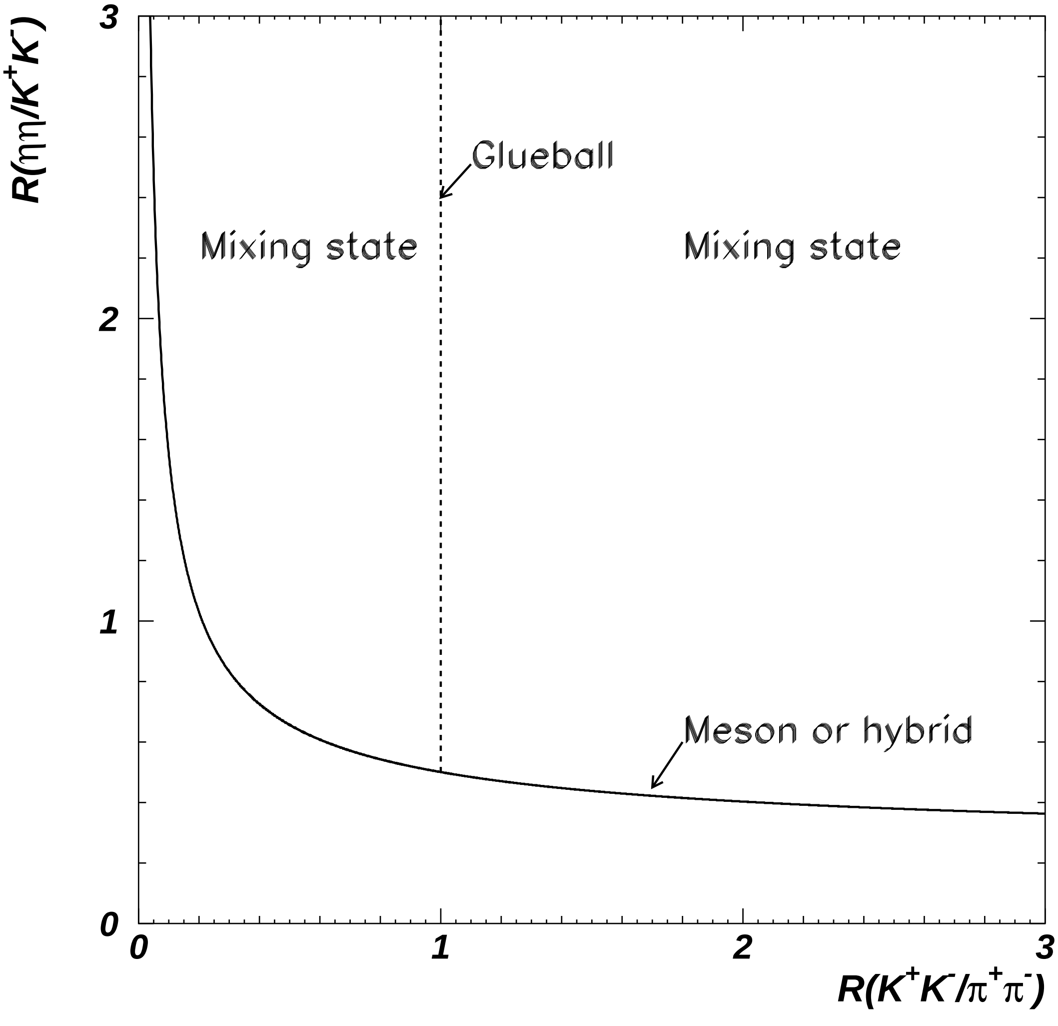

Figure 1: Discriminant via $R(\frac{\eta\eta}{K^+K^-})$ and $R(\frac{K^+K^-}{\pi^+\pi^-})$.

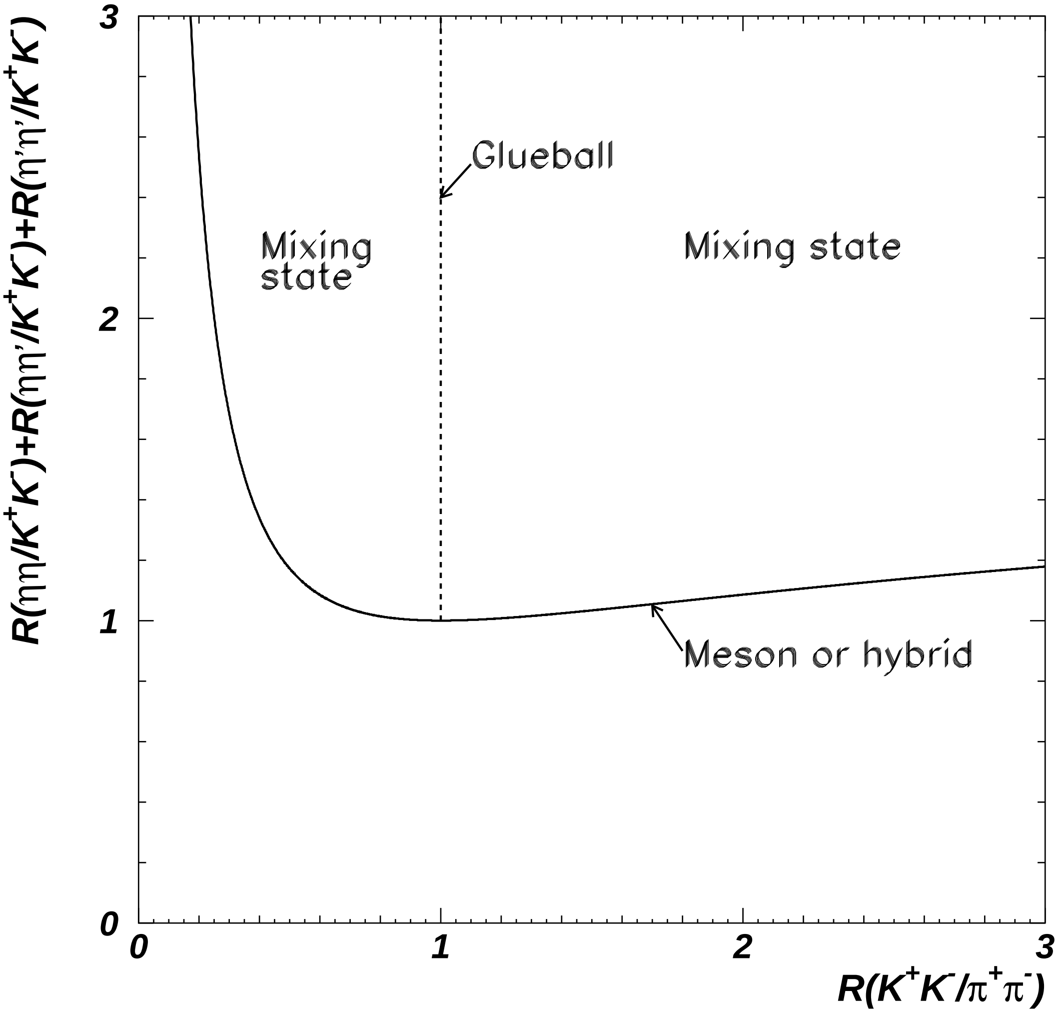

Figure 2: Discriminant via $R(\frac{\eta\eta}{K^+K^-}) + R(\frac{\eta\eta'}{K^+K^-}) + R(\frac{\eta'\eta'}{K^+K^-})$ and $R(\frac{K^+K^-}{\pi^+\pi^-})$.

two pseudoscalar decay modes are [4, 5]

$$Br(J/\psi \to \gamma f_J(2220))Br(f_J(2220) \to \pi^+\pi^-) = (5.6^{+1.8}_{-1.6} \pm 2.0) \times 10^{-5}, \tag{23}$$

$$Br(J/\psi \to \gamma f_J(2220))Br(f_J(2220) \to K^+K^-) = (3.3^{+1.6}_{-1.3} \pm 1.2) \times 10^{-5}, \tag{24}$$

$$Br(J/\psi \to \gamma f_J(2220))Br(f_J(2220) \to K^0_s K^0_s) = (2.7^{+1.1}_{-0.9} \pm 0.8) \times 10^{-5}, \tag{25}$$

This means that if the spin of $f_J(2220)$ is $J = 2$, then

$$R(\frac{K^+K^-}{\pi^+\pi^-}) = 0.98 \pm 0.72. \tag{26}$$

¿From (12) and (13) one may get discriminants that if $f_J(2220)$ is a pure meson or hybrid, the values of $R(\frac{\eta\eta}{K^+K^-})$ and $R(\frac{\eta\eta}{K^+K^-}) + R(\frac{\eta\eta'}{K^+K^-}) + R(\frac{\eta'\eta'}{K^+K^-})$ should be

$$R(\frac{\eta\eta}{K^+K^-}) = 0.50^{+0.40}_{-0.06} \tag{27}$$

and

$$R(\frac{\eta\eta}{K^+K^-}) + R(\frac{\eta\eta'}{K^+K^-}) + R(\frac{\eta'\eta'}{K^+K^-}) = 1.00^{+0.95}_{-0.00}. \tag{28}$$

¿From (19), (20) and (21) one may get discriminants that if $f_J(2220)$ is a pure glueball the values of $R(\frac{\eta\eta}{K^+K^-})$ and $R(\frac{\eta\eta}{K^+K^-}) + R(\frac{\eta\eta'}{K^+K^-}) + R(\frac{\eta'\eta'}{K^+K^-})$ should be

$$R(\frac{\eta\eta}{K^+K^-}) \gg 1.70 \tag{29}$$

and

$$R(\frac{\eta\eta}{K^+K^-}) + R(\frac{\eta\eta'}{K^+K^-}) + R(\frac{\eta'\eta'}{K^+K^-}) \gg 3.85. \tag{30}$$

Thus it is important to investigate the $\eta\eta$, $\eta\eta'$ and $\eta'\eta'$ decay modes of $f_J(2220)$ particle.

In summary, four discriminants based on the observed branching ratios of two pseudoscalar meson decay modes for a $I = 0, J^{PC} = even^{++}$ unflavored hadron X with mass between 1.2 GeV and 2.9 GeV can be used to judge whether X particle is a pure meson or hybrid, a pure glueball or a mixing state.

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